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## EQUIVALENCE PRINCIPLE AND RADIATION BY A UNIFORMLY ACCELERATED CHARGE

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We address the old question of whether or not a uniformly accelerated charged particle radiates, and consequently, if weak equivalence principle is violated by electrodynamics. We show that *radiation* has different meanings; some absolute, some relative. Detecting photons or electromagnetic waves is *not* absolute, it depends both on the electromagnetic field and on the state of motion of the antenna. An antenna used by a Rindler observer does not detect any radiation from a uniformly accelerated co-moving charged particle. Therefore, a Rindler observer cannot decide whether or not he is in an accelerated lab or in a gravitational field. We also discuss the general case.

Key words: equivalence principle, uniformly accelerated charge, relativity of radiation

### 1. INTRODUCTION

An accelerated charged particle radiates. Einstein's equivalence principle, on the other hand, tells us that the laws of physics in a constant gravitational field are the same as the laws of physics in an accelerated rocket. It is tempting therefore to conclude that a charged particle located on the table of our lab radiates. (Since according to the inertial freely falling observer, this charged particle is accelerating.) However, using the Maxwell's equations coupled to a static gravitational field, one can show that it does not radiate. It seems that either Einstein's equivalence principle is violated by electrodynamics, or a uniformly accelerated charged particle does not radiate.

The problem of radiation of a uniformly accelerated charged particle has an interesting and controversial history. In brief, based on a work by M. Born in 1909 [1], first W. Pauli [2] and then M. von Laue [3] gave arguments saying that such a charge does not radiate. Independently, G. A. Schott [4] derived the fields and concluded that such a charge radiates [5]. (His work was discussed in more detail by S. M. Milner [6].) Then in 1949, D. L. Drukey [7] published a short note in favour of radiation. In 1955, M. Bondi and T. Gold [8] have asserted that the Born solution did not treat the singularity of the potentials on the light cone correctly.

In 1960, T. Fulton and F. Röhrlich [9] published a paper discussing the problem in detail and concluding that: 1) "If the Maxwell-Lorentz equations are taken to be valid, and we consider retarded potentials only, and if radiation is defined in the usual Lorentz

invariant manner, a uniformly accelerated charge radiates at a constant non vanishing rate.” 2) “If one accepts the equations of motion based on the Abraham four-vector or on Dirac’s classical electrodynamics, the radiation reaction vanishes, but energy is still conserved.” 3) “a charged and a neutral particle in a homogeneous gravitational field behave exactly alike, except for the emission of radiation from the charged particle.” However, they argued, 4) “Radiation is defined by the behaviour of the fields in the limit of large distance from the source. Correspondingly, an observer who wants to detect radiation *cannot do so in the neighbourhood* of the particle’s geodesic.”

In 1963 F. Röhrlich [10] concluded that a freely falling observer in a static gravitational field with vanishing Riemann tensor, would see a supported charge radiating, and vice versa, i.e. a supported observer would see a freely falling charge also radiating. Röhrlich’s conclusions, which is in agreement with Einstein’s principle of equivalence, was re-derived by a different and more transparent method by A. Kovetz and G. E. Tauber in 1969 [11], and entered into the Problems of a text book by W. Rindler (problem 1.10 of [12]). In 1979, D. G. Boulware [13] wrote an article to show that “the equivalence principle paradox that the co-accelerating observer measures no radiation while a freely falling observer measures the standard radiation of an accelerated charge is resolved by noting that all the radiation goes into a region of space time inaccessible to the co-accelerating observer”.

In 1995, A. Singal [14] published a paper claiming again, and by a different method, that a uniformly accelerated charged particle does not radiate. This method and its conclusion is recently challenged by S. Parrott [15]. Parrott somewhere else [16] argued that “purely local experiments can distinguish a stationary charged particle in a static gravitational field from an accelerated particle in (gravity-free) Minkowski space”, in contradiction with Einstein’s principle of equivalence.

We see that, in addressing this problem some physicists argue that a uniformly accelerated charged particle does not radiate at all, while some other say that electrodynamics violates Einstein’s equivalence principle; and it seems that the problem is still not resolved. In this article, we show that this paradox is due to a misuse of the word *radiation*.

The scheme of the paper is as follows. In section II, we review the meaning of radiation. In section III, we address the problem of detecting radiation of a uniformly accelerated charge by a Rindler observer, and show that no radiation is detected, simply because no magnetic field is measured. In section IV, we show that this is due to a symmetry of the Minkowski spacetime: the symmetry with respect to Lorentz boosts, and we show that the same thing happens whenever the spacetime is static. We also show that in a stationary spacetime, there is an electromagnetic energy-like quantity, which is conserved for stationary charge distributions. In sections V and VI, we address the question of radiation in terms of the world line of the particle. There, we conclude that a freely falling charge, or a charge whose four-velocity is proportional to a static time-like vector in a static spacetime, does not radiate. Finally, section VII contains the concluding remarks.

## 2. RADIATION

The word *radiation* reminds us three concepts:

1. Flowing to infinity of energy in the form of electromagnetic waves.
2. Detection of photons by a suitable device such as a photographic plate or an

antenna.

3. Deviation of the world-line of a charged particle from that of a neutral one of the same mass experiencing the same force.

The first notion is the one which is always used to define radiation in standard text books. The second notion seems to lead to the following argument: If a system radiates, one can detect it by counting photons. The energy or momentum of these photons may differ for different observers but, since the number of them must be the same, we conclude that radiation is in some sense absolute, i.e., whether or not a system radiates does not depend on the observer. (See for example p. 506 of [9].) This argument, which is the basis for the paradox mentioned, is false. The important point is that radiation has different notions, some are absolute some are relative. (The absolute notion is, in fact, the third one.)

Consider a system of charged particles and the electromagnetic field produced by them. The divergence of the total energy-momentum tensor  $T^{\mu\nu} = T_e^{\mu\nu} + T_m^{\mu\nu}$  vanishes. If  $\xi^\mu$  is a Killing vector field,  $T^{\mu\nu}\xi_\mu$  is a conserved current. (Note that this is a local conservation law.)

Conservation of the corresponding quantity depends only on the system (through  $T^{\mu\nu}$ ) and the symmetry of the spacetime (through  $\xi^\mu$ ) and is not related to the observer or coordinate patch. In Minkowski spacetime the vector field  $\partial_t$  is a Killing vector field which is *everywhere* time-like. Gauss' theorem then, leads to the standard definition of radiation as flowing energy to infinity by electromagnetic waves. This is an *absolute* notion of radiation, which is closely related to the third concept of radiation as well. But we must note that for a curved spacetime this argument may fail. For example, there may be no time-like Killing vector field; or there may be an event horizon which prevents us to enclose the charges with an sphere at infinity; etc. These difficulties make it sometimes impossible to define radiation as *flowing energy to infinity*.

The second notion of radiation, viz. the detection of photons, is related to both the system and to the observer. Let  $u^\mu$  be the four-velocity of a local observer (not the source). This observer uses an apparatus to detect the electromagnetic flux of energy (or photons). Let  $\sigma^\nu$  be the space-like vector describing the direction of the apparatus used by him, and  $T_e^{\mu\nu}$  the energy-momentum tensor of the electromagnetic field. The amount of electromagnetic energy detected by the observer in proper time  $d\tau$  is proportional to the area  $d\Sigma$  of the apparatus used, and is equal to

$$dE = -T_e^{\mu\nu} u_\mu \sigma_\nu d\Sigma d\tau. \quad (1)$$

The quantity  $\varepsilon := -T_e^{\mu\nu} u_\mu \sigma_\nu$  is the flux of the electromagnetic energy through the surface  $d\Sigma$  of the apparatus, and depends both on the system and on the world-line of the observer.

Here it must be stressed that Fulton and Röhrlich's definition of radiation (referring to Synge [19]) is different [9, p. 506]. To define radiation they use the quantity

$$I = T^{\mu\nu} v_\mu^Q n_\nu \quad (2)$$

where  $v_\mu^Q$  is the velocity four-vector of the *source* of radiation. (A minus sign is not here because they use different conventions.) Then, they look at this quantity at the null infinity. In doing this, one must transport  $v_\mu^Q$  away from the source location. This can be done only in Minkowski (flat) spacetime. In curved spacetimes, this is not a well

defined quantity.

### 3. DISCUSSION OF THE PARADOX

Consider a charged particle  $S$  located inside a rocket, which is uniformly accelerated with respect to an inertial frame  $I$ . (Specifically, we mean a Rindler rocket; see pp. 49–51 and 156–157 of [12].) The question is whether or not the Rindler observer sees this charged particle radiating, i.e., whether or not he can detect photons. If he can, then he can deduce that he is inside an accelerating lab and not in a constant gravitational field. But we show that he cannot, and therefore, Einstein’s principle of equivalence is not violated. Concretely speaking, we show:

1. If the Rindler observer uses a local static antenna, he will not detect photons.
2. There is a conserved current of the form  $-T_e^{\mu\nu}\Xi_\mu$ , where  $\Xi$  is the generator of translation in Rindler time. It is quite natural to call the corresponding conserved quantity *the energy*.

The proof of these two statements is based on a symmetry of the Rindler spacetime. Rindler frame is given by the following 3 parameter family of time-like world-lines:

$$x^2 - t^2 = X^2, \quad y = Y, \quad z = Z. \quad (3)$$

For the spatial coordinate, the Rindler observer uses  $(X, Y, Z)$ , while for the time he uses the Lorentz boost parameter (or rapidity)  $\omega := \tanh^{-1} t/x$ . The proper time measured by a clock at  $(X, Y, Z)$  is  $\tau = \omega/X$ . The world-line of an observer located at  $(X, Y, Z)$  can be obtained by hyperbolic rotations in the  $(t, x)$  plane. Rindler time  $\omega$  is simply the hyperbolic angle of this rotation.

Now suppose there is a charged particle located at  $(X_s, Y_s, Z_s)$  and an antenna at  $(X_o, Y_o, Z_o)$ . These objects are moving in the Minkowski spacetime. Since the Green’s function for the 3+1 dimensional wave equation in Minkowski spacetime is non-zero only on the light-cone, we know that what  $O$  measures at a Rindler time  $\omega_o$  is due to the state of motion of  $S$  at the retarded time, i.e., at the intersection of the past light-cone of the event  $(\omega_o, X_o, Y_o, Z_o)$  with the world-line of  $S$ .

To write the Poynting vector at Rindler instant  $\omega_o$  for the local observer who is seated at  $(X_o, Y_o, Z_o)$ , we can write everything in the instantaneous rest frame of the source  $S$  at the retarded time and then use the Lorentz boost that transforms this frame to the instantaneous rest frame of  $O$  (at the moment of observation).

Since  $S$  is uniformly accelerated, its acceleration at its instantaneous rest frame is  $\alpha_s = 1/X_s$ . From electrodynamics we know that at the event  $(t_o, x_o, y_o, z_o)$ , the electromagnetic fields are given by

$$\mathbf{E} = q \left[ \frac{\hat{\mathbf{r}} - \mathbf{v}}{\gamma^2 (1 - \mathbf{v} \cdot \hat{\mathbf{r}}) r^2} \right]_{\text{ret}} + q \left[ \frac{\hat{\mathbf{r}} \times \{(\hat{\mathbf{r}} - \mathbf{v}) \times \dot{\mathbf{v}}\}}{(1 - \mathbf{v} \cdot \hat{\mathbf{r}})^3 r} \right]_{\text{ret}}, \quad (4)$$

$$\mathbf{B} = [\hat{\mathbf{r}} \times \mathbf{E}]_{\text{ret}}. \quad (5)$$

Here a hat means unit vector and the subscript *ret* means that everything must be computed at the retarded event  $(t_s, x_s, y_s, z_s)$ . Therefore, the electromagnetic field at the observation event is

$$\mathbf{E} = \frac{q}{r^2} + \frac{q\alpha}{r} \hat{\mathbf{r}} \times (\hat{\mathbf{r}} \times \hat{\mathbf{x}}), \quad (6)$$

$$\mathbf{B} = \frac{q\alpha}{r} \hat{\mathbf{x}} \times \hat{\mathbf{r}}. \quad (7)$$

It is important, however, to notice that these expressions are in the instantaneous rest frame of  $S$ . To obtain electromagnetic fields as measured by the observer  $O$ , we must use the Lorentz transformations

$$\mathbf{E}' = \gamma (\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{\gamma^2}{\gamma + 1} \mathbf{v} \mathbf{v} \cdot \mathbf{E}, \quad (8)$$

$$\mathbf{B}' = \gamma (\mathbf{B} - \mathbf{v} \times \mathbf{E}) - \frac{\gamma^2}{\gamma + 1} \mathbf{v} \mathbf{v} \cdot \mathbf{B}. \quad (9)$$

We have chosen the inertial coordinate system such that at the retarded event the source's velocity  $v_s = t_s/x_s$  vanishes; therefore, at the retarded event  $t_s = 0$ . The world-line of the source is  $x_s^2 - t_s^2 = X_s^2 = \alpha_s^{-2} = \text{constant}$ . From this it follows that the acceleration of the source at the retarded time is  $\alpha = 1/x_s$ .

From  $x_o^2 - t_o^2 = X_o^2 = \alpha_o^{-2} = \text{constant}$ , which describes the world-line of the antenna (i.e. local observer), it follows that  $v = t_o/x_o$ . We also note that  $r = (t_o - t_s) = t_o$  and  $\hat{\mathbf{x}} \cdot \mathbf{r} = x_o - x_s$ . From these ingredients, it is easy to see that

$$\mathbf{B}' = 0. \quad (10)$$

This shows that a local Rindler observer  $O$ , whose world-line is given by

$$x_o^2 - t_o^2 = X_o^2, \quad y_o = Y_o, \quad z_o = Z_o, \quad (11)$$

sees a pure electric field and, therefore, no radiation.

It is worthy of mention that Pauli's argument, based on the fields derived by Born, is also the vanishing of the magnetic field. However, as mentioned by Bondi and Gold [8], and Fulton and Röhrlich [9], the basis of his derivation is not true. Here again we see that the magnetic field vanishes for the (Rindler) co-moving observer, and we see this by exactly computing the fields as measured by this observer.

The argument given above depends deeply on a symmetry of the Minkowski spacetime, viz. the existence of the time-like Killing vector field  $t\partial_x + x\partial_t$ , which for the Rindler observer is just  $\partial_\omega$ . In the next section we discuss the general case.

#### 4. RADIATION IN TERMS OF THE ELECTROMAGNETIC ENERGY-MOMENTUM TENSOR

A stationary spacetime is a spacetime with a time-like Killing vector  $\xi := \partial/\partial t$ . In such a spacetime, one can choose a coordinate system in which, the metric components are  $t$ -stationary charge distribution  $J^\mu$  in a stationary spacetime:  $\partial_0 J^\mu = 0$ , and  $\partial_\mu (\sqrt{|g|} J^\mu) = \partial_i (\sqrt{|g|} J^i) = 0$ . Using a  $t$ -independent ansatz for the four-potential  $A_\mu$ , it is easily seen that the field-strength tensor  $F_{\mu\nu}$  is  $t$ -independent. Moreover, the source-full Maxwell equation

$$\frac{1}{\sqrt{|g|}} \left( g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta} \sqrt{|g|} \right)_{,\nu} = J^\mu, \quad (12)$$

shows that there is no inconsistency in taking  $F_{\mu\nu}$  independent of  $t$ , because both  $j^\mu$  and  $g_{\mu\nu}$  are  $t$ -independent. In fact, if  $F_{\mu\nu}(t, \mathbf{r})$  is a solution to (12),  $F_{\mu\nu}(t + \Delta, \mathbf{r})$  is also

a solution. So, if the Maxwell equations have a unique solution in this spacetime, the field-strength tensor should be  $t$ -independent.

Since  $\xi$  is a Killing vector, we have

$$(-T^{\mu\nu}\xi_\nu)_{;\mu} = \frac{1}{\sqrt{|g|}} \left( -\sqrt{|g|}T^\mu_0 \right)_{;\mu} = 0. \quad (13)$$

So, a conserved current  $\mathcal{J}^\mu := -T^\mu_0$  exists. One can define a corresponding current for the electromagnetic part of the energy-momentum tensor. However, as  $T_e^{\mu\nu}$  is  $t$ -independent, it is seen that

$$\int_\Sigma (-T^\mu_0)_{\text{em}} \sqrt{|g|} dS_\mu = \int_{\Sigma'} (-T^\mu_0)_{\text{em}} \sqrt{|g|} dS_\mu, \quad (14)$$

where  $\Sigma$  is any hyper-surface, and  $\Sigma'$  is the hypersurface formed by translating  $\Sigma$  along the Killing vector field  $\xi$  by some value  $\Delta$ . This means that the energy-like quantity of the electromagnetic field in any hypersurface (any portion of space), including or excluding the charge(s), does not change. In this sense, one can say that a stationary charge distribution in a stationary spacetime does *not* radiate.

A static spacetime is a stationary spacetime, for which there exists a family of space-like hyper-surfaces normal to the time-like Killing vector field  $\xi$ . This means that there exists a suitable choice of coordinates, for which  $g_{0i} = g^{0i} = 0$ , and  $g_{\mu\nu}$ 's are all  $t$ -independent. In such a spacetime, consider a static charge distribution. A static charge distribution is a stationary one with the additional condition  $J^i = 0$ . For the field produced by a static distribution one can take the ansatz:  $A_i = 0$  and  $\partial_0 A_0 = 0$ . From this, and the fact that the metric is  $t$ -independent and block-diagonal, it is easy to see that

$$F_{ij} = F_i{}^j = F^{ij} = 0. \quad (15)$$

This shows that the source-full Maxwell equation is identically satisfied for  $\mu \neq 0$ . For  $\mu = 0$ ,

$$-\frac{1}{\sqrt{|g|}} \left( g^{00} g^{ij} \sqrt{|g|} \partial_j A_0 \right)_{;i} = J^0. \quad (16)$$

This equation is consistent, since the left-hand side is  $t$ -independent, as well as the right-hand side.

This solution to the static charge distribution has no magnetic field, and has a  $t$ -independent electric field. By magnetic field (in a covariant form) we mean the tensor  $B_{\mu\nu} := F_{\alpha\beta} h^\alpha_\mu h^\beta_\nu$ . Here  $h_{\mu\nu}$  is the projector normal to  $\xi$ , that is  $h_{\mu\nu} := g_{\mu\nu} - (\xi_\mu \xi_\nu) / (\xi \cdot \xi)$ . Note that for the choice of coordinates introduced above,  $h_{ij} = g_{ij}$ ,  $h_{00} = h_{i0} = 0$ ,  $B_{ij} = F_{ij}$ , and  $B_{0i} = 0$ . The electric field is similarly defined through  $E_\mu := F_{\mu\nu} \xi^\nu / \sqrt{-\xi \cdot \xi}$ .

From (15), we have  $-T^i_{e0} = -h^\mu_\nu T^\nu_e \xi^\alpha = 0$ . Consider an observer with the four-velocity  $u^\mu = \xi^\mu / \sqrt{-\xi \cdot \xi}$ . For this observer the magnetic field is just  $B_{\mu\nu}$ , and the electric field is  $E_\mu$ . So, this observer measures a  $t$ -independent electric field and no magnetic field. Moreover, what this observer measures as the Poynting vector is

$$\begin{aligned} S^\mu &:= -u_\nu T^{\alpha\nu} h^\mu_\alpha \\ &= -u_\nu F^{\beta\nu} F_\beta{}^\alpha h^\mu_\alpha \\ &= E^\beta B^\mu{}_\beta, \end{aligned} \quad (17)$$

which is identically zero. This means that, in terms of the Poynting vector, this observer measures no radiation, simply because the magnetic field is zero for this observer. This conclusion is stronger than that of the last subsection. That meant no net flux of the electromagnetic *energy* is observed by the stationary observers. This means that, besides, no electromagnetic *energy* current is observed by such an observer.

## 5. RADIATION AND THE WORLD-LINE OF CHARGED PARTICLES

Is it true that, in a gravitational field, charged particles fall the same as uncharged particles? This problem is also related to the paradox mentioned at the beginning of this paper.

Of course, this question must be answered experimentally (and the experiment is more difficult than it seems, cf. [18].) But let us study the answer given by the known theory of electrodynamics. The question is not trivial, for acceleration causes radiation and this may cause a damping. It seems therefore that a charged particle does not fall the same as an uncharged particle. To answer this question we have to know the form of the radiation reaction force. The best candidate for this, is the Lorentz-Abraham-Dirac force from which it follows that charged particles fall the same as neutral particles, i.e. the weak equivalence principle is fulfilled. To our knowledge, this was first noticed by Röhrlich [10]. Here, we present a heuristic argument in favour of the conclusion that equivalence principle is not violated by charged particles.

To begin with, let us consider a stationary charged particle in the Minkowski spacetime. The world-line of such a particle is

$$-\infty < t_s < \infty, \quad x_s = \text{const.} > 0, \quad y_s = z_s = 0. \quad (18)$$

A Rindler observer sees only the segment  $-x_s < t < x_s$ ; by the following world-line:

$$-\infty < \omega_s < \infty, \quad X_s = x_s / \cosh \omega_s, \quad Y_s = Z_s = 0. \quad (19)$$

Trivially, this world-line is a geodesic describing the motion of the particle approaching the horizon as  $\omega \rightarrow \pm\infty$ . Now let's interpret the Rindler spacetime as a gravitational field. Since we know that in the Minkowski spacetime the charged particle follows a geodesic, a little reflection shows that in the Rindler gravitational field a free charged particle falls the same as a free uncharged particle. To this problem, let's apply Einstein's equivalence principle. In the comoving freely falling lab, which is a local inertial frame, the charged particle is at rest and therefore it does not radiate. The comoving observer sees no reason for the charged particle to move in the lab, simply because it is completely free. Therefore, with respect to the freely falling lab, the charged particle is always at the same position. Transforming this result to the Rindler frame, we conclude that in the gravitational field of the Rindler spacetime, charged particles fall the same as the uncharged particles. The electromagnetic field produced by this charged particle as seen by the inertial comoving observer is purely electric, a Rindler observer, however, sees also a magnetic field and a non-vanishing Poynting vector; and sees that the charged particle goes to the horizon  $X = 0$  as  $\omega \rightarrow \infty$ . If the Rindler observer uses a local device, such as a camera, he will observe some photons, i.e. he will receive some energy which causes an effect on his photographic plate. This effect, however, is not radiation. To convince, suppose a charged particle is in uniform rectilinear motion relative to an inertial observer. If this observer uses an antenna, his antenna does receive some energy,

simply because the Poynting vector at the position of antenna is non-zero (and it is even time varying). However, this is simply the result of an interaction of antenna with the moving charge, (and of course, as a result of this interaction the charged particle's trajectory is affected).

In the previous sections, it was shown that a uniformly accelerated charge in a Minkowski spacetime does *not* radiate, in the sense that for the Rindler observer the Poynting vector vanishes, and an energy-like quantity for the electromagnetic field is constant. This means that, according to Rindler observers, no extra force is needed to maintain the uniform acceleration of such a charged particle (of course no extra force beside the force needed for a neutral particle of the same mass to have that acceleration). In other words, the world-line of the charged particle will be the same as that of a neutral particle.

Now consider a Rindler gravitational field, in which a charged particle is stationary. One can obtain the electromagnetic field of this particle in a manner exactly the same as that of section III, which shows that there is no radiation.

Do, in some sense, these two problems differ? Some authors [16] argue that to support a charged particle in a gravitational field a rocket is needed, and this rocket spends more fuel than a rocket needed to do the same thing for a neutral particle.

However, the results of previous sections, in terms of the electromagnetic energy, make no difference between charged and neutral particles in any of the two cases. According to Rindler observers, no extra force is needed for the charge, and as the four-vector of force is zero according to one observer, it should be zero according to other observers as well. So the result of the above gedanken experiment should be null.

Another problem is that of a freely falling charge in a Rindler gravitational field. Such a charge moves uniformly according to Minkowski observers, so that it does not radiate according to them, and its world-line should be the same as that of a neutral particle. In fact, this problem, in terms of energy considerations, is the same as the problem of a stationary (or uniformly moving) charge in Minkowski spacetime. These results are also in agreement with Einstein's equivalence principle.

## 6. RADIATION REACTION FORCE

We show that these results are also true in terms of the damping force, so that there is no rocket paradox. We show that this force is zero in certain cases, which are the generalisations of the above cases.

When a particle radiates, it experiences a force due to its radiation. (The original derivation of the reaction force is due to Dirac [17]. A recent derivation is given by A. Gupta and T. Padmanabhan [20].) The relativistic form of the Abraham-Lorentz-Dirac force, experienced by an accelerated charge is

$$f_{\text{ALD}}^\mu = \frac{e^2}{6\pi} \left( \frac{D^2 u^\mu}{D\tau^2} + u^\mu u_\alpha \frac{D^2 u^\alpha}{D\tau^2} \right), \quad (20)$$

where  $e$  is the charge of the particle,  $u$  is its four-velocity, and  $D/D\tau$  is covariant differentiation with respect to the proper time  $\tau$ . To obtain this self-force, it is assumed that the power radiated by an accelerated charge, in its instantaneous rest frame, is

$$P = \frac{e^2}{6\pi} \frac{d\mathbf{v}}{dt} \cdot \frac{d\mathbf{v}}{dt}, \quad (21)$$

which is proved by calculating the amount of the electromagnetic energy escaping to



the infinity, for a *localised* accelerated charge in the Minkowski spacetime. Moreover, to obtain (20) from (21), an integration by part is needed, which is also valid (the boundary terms vanish) provided the charge is *localised*. Neither (21), nor (20) can be proved for non-localised accelerated charges. However, if one assumes that the self-force experienced by a particle is *local*, that is, it depends only on the status of the charge in an infinitesimal neighbourhood of an instant, one can generalise (20) for the case where the charge is not localised, or the spacetime is not Minkowskian. This does *not* necessarily mean that (21), or its relativistic generalisation

$$P = \frac{e^2}{6\pi} \frac{Du}{D\tau} \cdot \frac{Du}{D\tau}, \quad (22)$$

are also valid in these more general cases.

Now consider the Abraham-Lorentz-Dirac force in two special cases:

**A-** A charge, the four-velocity of which is proportional to a static time-like Killing vector field, i.e., a Killing vector field for which there exists a family of hyper-surfaces normal to it. It is obvious that this situation may only happen in a static spacetime. In this case,  $u^0 = 1/\sqrt{-\xi \cdot \xi}$ , and  $u^i = 0$ . Also note that  $d(\xi \cdot \xi)/d\tau = 0$ , and that in a static spacetime  $\Gamma^0_{00} = \Gamma^i_{0j} = 0$ , which shows that

$$\frac{Du^0}{D\tau} = 0, \quad (23)$$

$$\frac{Du^i}{D\tau} = \Gamma^i_{00} (u^0)^2, \quad (24)$$

and

$$\frac{D^2 u^i}{D\tau^2} = (u^0)^2 \Gamma^i_{0j} \frac{Du^j}{D\tau} = 0. \quad (25)$$

Therefore

$$f_{\text{ALD}}^\mu = 0. \quad (26)$$

So this charge experiences no self-force, even though its four-acceleration may be nonzero (if  $\xi \cdot \xi$  is space-dependent). In other words, the force needed to accelerate a charge to this specific four-velocity ( $u^0 = 1/\sqrt{-\xi \cdot \xi}$ , and  $u^i = 0$ ) is the same as the force needed to accelerate an uncharged particle of the same mass. This is in agreement with the conclusion of section IV: “An accelerated charge distribution does not radiate according to certain observers, whenever the charged particles and the observers move along the same static Killing vector field.” The case of a uniformly accelerated charge in the Rindler rocket, discussed in section III is a special case.

**B-** A freely falling charge, in an *arbitrary* spacetime. In this case, the four-acceleration is zero ( $Du^\mu/D\tau = 0$ ) which shows that the self-force is zero. Note that in a general spacetime, it may be impossible to define an energy-like quantity, since there may be no time-like Killing vector field. The above conclusion in a sense shows that a freely falling charge does not radiate; in the sense that the world-line of a freely falling charged particle is the same as that of an uncharged particle.

## 7. CONCLUSIONS

The notion of radiation in terms of receiving electromagnetic energy by an observer is not absolute, but this relative notion is consistent with the principle of equivalence.

That is, in a static spacetime, a supported charge does not radiate according to another supported observer; neither does a freely falling charge according to a freely falling observer. Also, a freely falling charge does radiate according to a supported observer, and a supported charge does radiate according to a freely falling observer.

The absolute meaning of radiation, i.e. radiation according to world line of the charge, was also discussed. We saw that a supported charge in a static spacetime, or a freely falling charge, do not radiate, in the sense that no extra force is needed to maintain their world-line the same as that of a neutral particle.

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